Unique Paper Code : **62354343**

Name of the Course : **B.A. (Prog.) Mathematics**

Name of the Paper : **Analytic Geometry and Applied Algebra**

Semester : **III (CBCS)**

Time : 3 Hours **Maximum Marks :75**

- **Attempt any four questions in all.**
- **All questions carry equal marks.**

1. (a) Identify and sketch the curve:

$$y = 4x^2 + 8x + 5$$

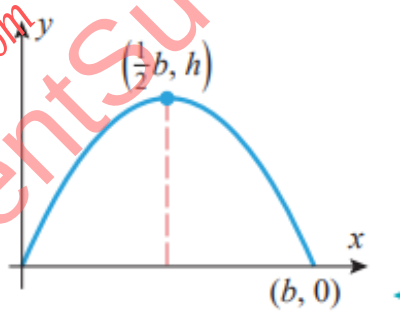
Also label the focus, vertex and directrix

(b) Describe the graph of the curve:

$$x^2 + 9y^2 + 2x - 18y + 1 = 0$$

Find its foci, vertices and the ends of the minor axis.

(c) Find an equation for the parabolic arch with base b and height h, shown in the accompanying figure



2. (a) Find the equation of parabola with vertex (2,4) and focus (3,4).

(b) Find the equation for the ellipse that has ends of major axis (±6, 0) and passes through (2, 3)

(c) Find the equation for a hyperbola that satisfies the given conditions:
Asymptotes y = 2x + 1, y = –2x + 3 and passes through the origin.

3. (a) Find an equation of the sphere with centre (2,-1,-3) and satisfying
i) Tangent to the x-y plane.

ii) Tangent to the x-z plane.
iii) Tangent to the y-z plane.

b) Show that the graph of the equation:

$$\sqrt{x}+\sqrt{y}=1, \quad \forall x \in [0,1], y \in [0,1]$$

is a portion of a parabola.

(c) Describe the surface whose equation is given as

$$x^2+y^2+z^2+2y-6z+5=0$$

4. (a) Find $\vec{u}$ and $\vec{v}$ if $5\vec{u}+2\vec{v}=6\hat{i}-5\hat{j}+4\hat{k}$ and $3\vec{u}-4\vec{v}=\hat{i}+2\hat{j}+9\hat{k}$ Also find a vector of length 3 and oppositely directed to $\vec{v}$.

(b) (i) Find the projection of $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$ on $\vec{b}=\hat{i}-2\hat{j}+\hat{k}$

(ii) Determine whether $\vec{u}=<6, 1, 3>$ and $\vec{v}=<4, -6, -7>$ make an acute angle, an obtuse angle or are orthogonal? Justify your answer.

(c) Find the volume of the parallelopiped with adjacent edges $\vec{u}=3\hat{i}+2\hat{j}+\hat{k}$, $\vec{v}=\hat{i}+\hat{j}+2\hat{k}$ and $\vec{w}=\hat{i}+3\hat{j}+3\hat{k}$. Also find the area of the face determined by $\vec{u}$ and $\vec{v}$.

5. (a) Find the distance of the point P(2, 5, −3) from the plane

$$\vec{r}.(6\hat{i}-3\hat{j}+2\hat{k})=4$$

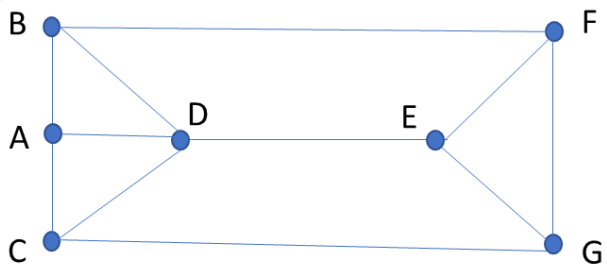
(b) Find the equation of the plane through the points $P_1(2, 1, 4)$, $P_2(1, 0, -3)$ that is perpendicular to the plane $4x + y + 3z = 2$.

(c) Show that the lines L_1 and L_2 are parallel and find the distance between them

$$L_1: x=2-t, \quad y=2t, \quad z=3+t$$
$$L_2: x=-1+2t, \quad y=3-4t, \quad z=5-2t$$

6. (a) Suppose a job placement agency wants to schedule interviews for candidates Ann, Judy and Carol with interviewers Al, Brian and Carl on Monday, Tuesday and Wednesday in such a way that each candidate gets interviewed by each interviewer. Solve this problem using a Latin Square.

(b) Find a vertex basis for the following graph:



(c) For the following graph, find a minimal edge cover and a maximal independent set of vertices.